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Stochastic assessment of steel rope strength using magnetic NDT results

1. Introduction

The magnetic flux detector Intros detects two types of steel ropes damage: localised faults (LF) such as wire breaks and faults distributed along the rope length such as the loss of cross-sectional metallic area (LMA) (Mironenko & Sukhorukov, 1998). The measured characteristics of local and distributed defects are integral indexes of degradation. They do not account for the broken and missing wire distribution over the rope strands, wire layers in each strand and identical wire groups in every layer. Furthermore, both damage types have in general random locations. So the problem arrises to develop a technique for degraded rope strength estimation, which is based on magnetic NDT results and makes use of a probabilistic approach. In the absence of data concerning the mutual destructive action of LF and LMA on the rope's strength these two failure types will be considered separately. The combined effect may be estimated as a rough approximation by superposition.

2. Concept and basic relationships for LMA damaged ropes

First we will consider only the LMA influence estimate on the degraded rope endurance (residual strength). Endurance parameter η_{LMA} is treated as a percentage ratio of damaged (n_{LMA}) and non-damaged (n) ropes safety factors under the same loading conditions

$$\eta_{LMA} = \frac{n_{LMA}}{n} 100\% . \qquad (1)$$

As an example the straight six strand steel rope 6x19(9/9/1) FC is taken. Suppose the rope to be in pure tension state when the generalised longitudinal force and reactive torque appear at cross-sections. The own weight of the rope is ignored.

The steel wire rope theory (Glushko, 1966) is considered as a background of the rope strength assessment. The rope mechanical state equations connect an axial force T and torque H with relative elongation ε and relative angle of twist θ:

$$\left.\begin{aligned} T &= C_{11}\varepsilon + C_{12}\theta \\ H &= C_{12}\varepsilon + C_{22}\theta \end{aligned}\right\}, \qquad (2)$$

where C_{11}, C_{12} and C_{22} are the effective stiffness parameters of the rope as a heterogeneous structure. They depend on wires stiffness coefficients and geometric parameters of wires and strands helixes.

Hereinafter the term “group of wires” refers to the set of strand wires having the same diameter and the same lay parameters. The rope strands under consideration include three groups of wires: centre wire, inner wire layer and outer wire layer.

Let the following parameters be defined:

M number of strands;

β strand lay angle;

J number of wire groups in the strand;

α_j lay angle of j-th wire group relative to the strand lay axis;

r_j wire lay radius of j-th group;

D rope outer diameter;

d strand outer diameter;

$R=(D-d)/2$ – strand lay axis radius;

EA_j axial rigidity of one wire at j-th group;

k core compliance.

The number of wires in the m–th strand is equal to $N_m = \sum_{j=1}^{J} N_{m,j}$, where $N_{m,j}$ is the number of wires in j-th group in m-th strand.

Under pure tension the angle of twist is zero, hence the twist stiffness B is ignored in our study. Longitudinal stiffness C_{11} and longitudinal-twist stiffness C_{12} of non-damaged rope are defined by formulae:

$$C_{11} = \sum_{m=1}^{M} a_m \left(\cos^3 \beta - \mu_m \sin^2 \beta\right) + \frac{c_m}{R} \sin^2 \beta \left(2\cos^2 \beta + \mu_m \cos 2\beta\right), \qquad (3)$$

$$C_{22} = \sum_{m=1}^{M} a_m R \sin \beta \left(\cos^2 \beta - \mu_m \sin^2 \beta\right) + c_m \cos \beta \left[\left(1 + tg^4 \beta\right) \cos^4 \beta - \mu_m \cos 2\beta \sin^2 \beta\right]$$

Here $a_m = \sum_{j=1}^{J} N_{m,j} EA_j \cos^3 \alpha_j$ is the m-th strand longitudinal stiffness relative to its lay axis; $c_m = \sum_{j=1}^{J} N_{m,j} EA_j r_j \cos^2 \alpha_j \sin \alpha_j$ is the m-th strand longitudinal-twist stiffness relative to its lay axis; μ_m is the “Poison” coefficient of m-th strand depending on core compliance k and stiffness parameters that define the force along a principal axis normal to the strand lay axis.

Stress values in wires are calculated step by step. First, the generalised strain ε is determined from the system (2) for given load T and known stiffness parameter C_{11}. Due to consistent deformation conditions this strain serves as the elongation strain of each strand relative to the rope axis. After its transformation to the strand lay axis it is possible to evaluate tensile, bending and torsion strains and stresses in individual wires in their helix co-ordinate systems. Then safety factors with the aid of proper yield criterion for all of wire groups are found. The safety factor for the most loaded wire is assumed to be the required rope safety factor n.

The same procedure may be carried out to calculate the safety factor n_{LMA} for the damaged rope. One only needs to evaluate the stiffness parameter C_{11} for the prescribed distribution of wire loss related to measured value of LMA (Volokhovsky et al., 2001). If the wear picture is unknown the Monte-Carlo method (Sobol', 1973) seems to be the most suitable for a rather complicated rope design.

3. Strand wear picture simulation

To obtain the probability strength assessments of LMA damaged ropes from magnetic NDT results the following approach is proposed. A safety factor of damaged rope is calculated by the foregoing procedure that includes the statistical modelling of wear (loss of wires) locations in the strand cross-section. The simulation algorithm accounts for strand design features and previous information about strands deterioration specifics. For all that, samples characterise the random location of loss wires in cross-section. In the simulation process the scheme "sampling without return" is used (a lost wire cannot appear again). Furthermore, hypotheses on the distribution law of the initial fracture probabilities in rope strands are introduced. The simplest hypothesis refers to equal-probability distribution of wire damage. I.e. that initial probabilities of different wires loss P_i $(i = 1,...,N)$ are equal: $P_i = 1/N = const$. Here N is the number of wires in the non-damaged rope. Taking account of more probable fracture of thin (small) or, vice versa, thick (big) wires would be possible by means of weight coefficients, which reflect the specific physical fracture mechanisms. For example, from the fracture mechanics viewpoint bigger wire loss is more probable because of a relatively greater quantity of small-scaled initial defects. If corrosion or fretting-wear deterioration is dominant, a greatest rate of small wire loss seems to be more probable.

Thus, measured LMA value (denoted further by ΔA) and initial wire failure probabilities P_i are the simulation algorithm input parameters. Let $\tilde{N}_{m,j}$ denote the random number of remaining (not lost) wires in *j*-th wires group in *m*-th strand. To define stiffness coefficients of damaged rope one must substitute random values $\tilde{N}_{m,j}$ for non-damaged rope parameters $N_{m,j}$ in the equations (3). Then, the maximum equivalent stresses in the remaining wires are calculated and the rope residual endurance (random value) η_{LMA} is found. After that, with use of standard statistic procedures, one builds rope endurance and lost wires quantity histograms for every deterioration type, determines mean values assessments and corresponding assessment errors for given confidence probability.

4. Results for LMA-endurance calculations

The procedure outlined above was applied to the rope 6x19(9/9/1) FC with the following parameters: $M = 6$, $J = 3$, $\beta = 16.5°$, $D = 22.5\times10^{-3}$ m, $d = 7.5\times10^{-3}$ m, $E = 2\times10^{11}$ Pa, $k = 0.2\times10^{-7}$ m/N. Diameters of centre wire, wires at inner and outer strand wire layers were $0.2788d$, $0.1322d$ and $0.2478d$ respectively; wire lay angles for ordinary lay rope were 0°, 9°10′, –16°27′. Cross-section loss ΔA was taken equal to 16%. The number of realisations in every initial failure probability variant was set to 500.

Histograms of rope endurance (residual strength) and wire loss quantity for three variants of initial failure probabilities P_i are shown in Figure 1. The diagrams in Figure 1a correspond to the equal initial probabilities, in Figure 1b - to initial probabilities proportional to the wires cross-section squares A_i, and in Figure 1c - inversely proportional to wires cross-section squares A_i. The rope endurance has the least value when the scale strength effect $P_i \sim A_i$ is accepted in the model. Monte-Carlo procedure allows revealing relatively non-favorable and relatively favorable wear distribution through the cross-section (rare events, which correspond to low and high endurance estimates), and also wear distribution corresponding to expected estimate (typical event). The expected estimate errors with confidence probability 0.997 for the results in Figures 1a, 1b and 1c are equal to 0.135, 0.12 and 0.13% correspondingly.

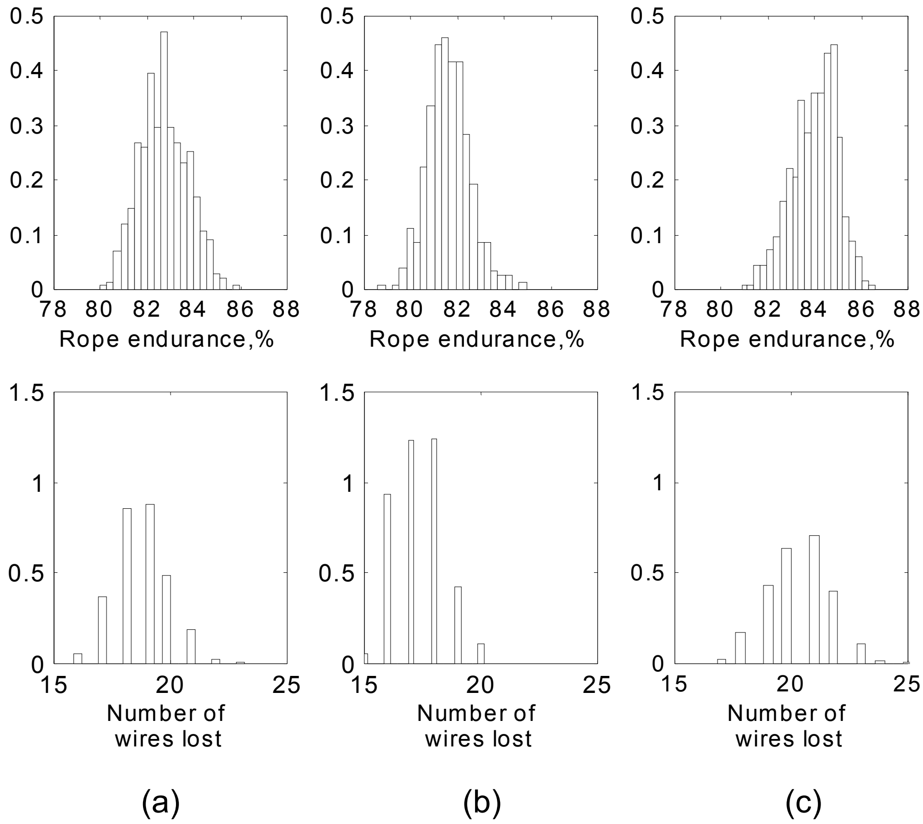

Figure 1: Histograms of damaged rope parameters for different initial failure probabilities of the wires: $P_i = const(a), P_i \sim A_i(b), P_i \sim 1/A_i(c)$.

5. Concept, basic relationships and results for LF damaged ropes

Consider now the model for prediction of the influence of localised faults (LF) of such as wire breaks on rope endurance loss. Experiments indicate that a broken wire in ordinary lay rope begins to bear the same load as non-broken wires nearly three lay lengths from the break location due to the friction. The simple scheme of how the broken wire takes gradually the load with moving away from break location was proposed by Malinovsky (2001). We use his concept to determine the friction "influence function" relative to single broken wire endurance loss. The rope lay length h may be estimated as $h \approx 6D$ where D is a rope diameter. The afore mentioned rope has a lay length equal to $h \approx 135$ mm. Let ξ denote a coordinate of LF (single wire break), and x denote a coordinate of wire (and rope cross-section as a whole) where that single wire endurance loss and thus induced rope endurance loss are estimated. The friction influence function at the single break vicinity $G(x-\xi)$ may be introduced as:

$$G(x-\xi) = \begin{cases} 0 \quad where \quad |x-\xi| \geq h, \\ \phi(x-\xi) \quad where \quad |x-\xi| < 3h. \end{cases}$$

Here $\phi(x-\xi)$ is the empirical function of broken wire endurance loss from Figure 2.

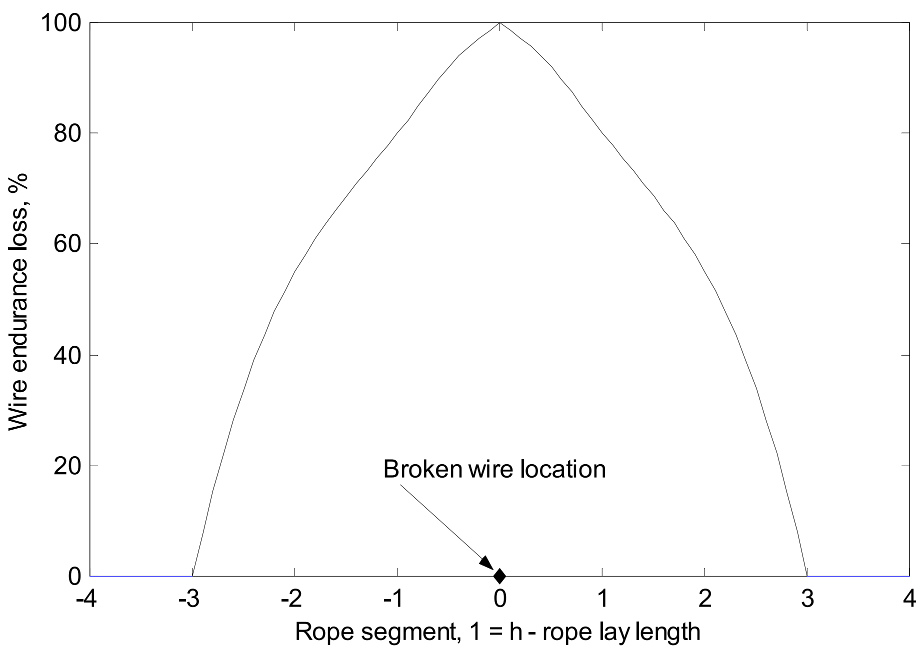

Figure 2: Influence function of wire break.

Let L denote the number of local faults on the rope length to be inspected. Percent measure of contribution of these plural defects to rope endurance loss at the cross-section x may be roughly expressed by the value:

$$\chi_{LF}(x) = \frac{1}{N}\sum_{l=1}^{L} G(x - \xi_l), \tag{4}$$

where ξ_l are coordinates of break locations, N is a total number of wires in non-damaged rope. More accurate assessment takes account of the rope strength degradation because of wire breaks:

$$\chi_{LF}(x) = \sum_{l=1}^{L}\left(1 - \frac{n_{LF}^{(l)}}{n}\right) G(x - \xi_l). \tag{5}$$

Here $n_{LF}^{(l)}$ is safety factor of the degraded rope with l-th broken wire calculated in cross-section $x = \xi_l$ by foregoing procedure, n is the safety factor of the un-damaged rope.

Table 1 shows results concerning rope endurance loss χ in section $x = \xi$ with wire break for different strand wire groups: centre one, inner and outer layers. Data are obtained by refined theory (Volokhovsky *et al.*, 2001), which accounts for tension, bending and torsion of wires, and with formula $\chi_{LF} = \Delta A$ (elementary prediction), where ΔA is the percent loss of metallic cross-section area as a result of a wire break.

Calculation model	Centre wire	Inner layer wire	Outer layer wire
Refined theory	2.42	0.52	1.69
Elementary formula	1.64	0.37	1.30

Table 1: Rope endurance loss at section with single wire break.

Steel rope safety standards for different types of lifting machines permit something like twice the number of wire breaks over $5h$-length than on h-length (RD ROSEK 012-97,1997). Let us verify this criterion numerically considering wire breaks in the outer strand layers, which can be counted most reliably by visual inspection or by magnetic flux detector. Local faults distribution along the rope axis has a random character. So a statistical assessment of wire breaks influence on the rope endurance loss will be carried out. Suppose wire break coordinates follow the uniform distribution law on normative lengths h and $5h$ symmetrically to cross-section $x = 0$. As an example, let such a typical value as 5 breaks locate on one lay length h and *twice more* (10 breaks) – on $5h$ length.

Estimates of rope endurance loss $\chi_{LF}(x)$ evaluated with the use of equation (5) over two hundred iterations are shown in Figure 3. Points mark the mean values; and upper and lower lines correspond to confidence limits with assessment reliability 0.998.

Strength loss appears more significant for the double breaks number on $5h$-length segment than on one lay length h. This tendency is observed also for breaks of inner layer wires and centre wire. Hence, $5h$-length safety criterion seems to be an unduly optimistic one, if, furthermore, a possible distributed cross-sectional loss is accounted as an additional non-favourable factor. Maximum values of curves for both safety criteria become nearly the same if breaks number on $5h$-segment is equal to 1.4 of the *h* segment breaks number. This result also is valid for inner layer wires and for centre wire. In that way, one of normative criterion related to local rope fault (LF) distribution needs refining.

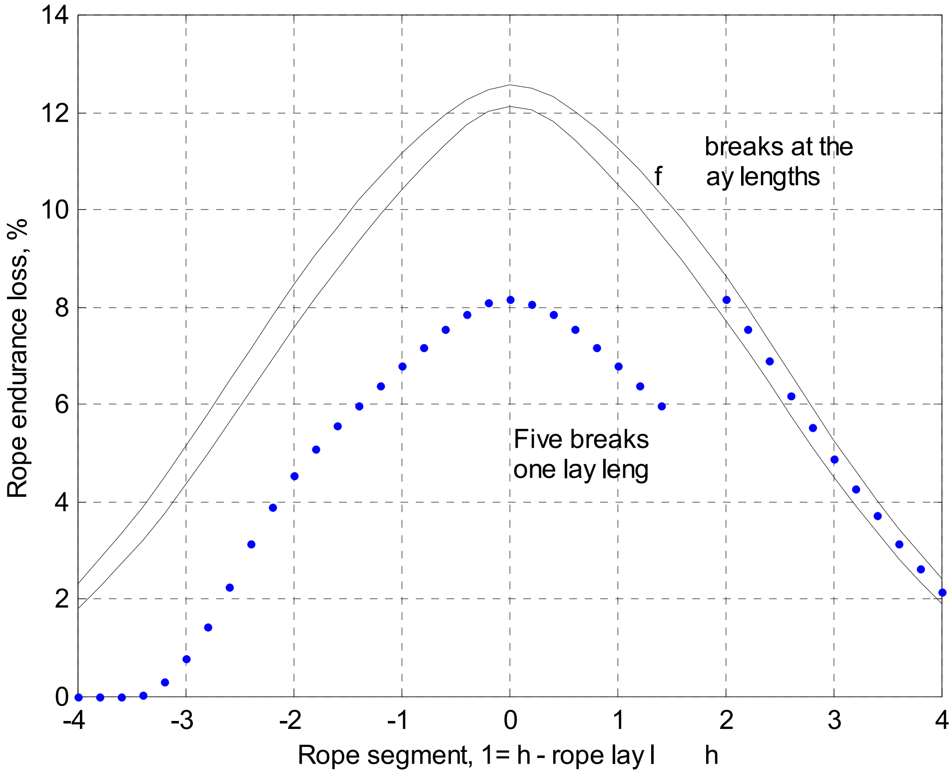

Figure 3: Statistical assessment of steel rope endurance loss due to wire breaks.

6. Both LMA & LF damaged ropes

Setting aside the correlation between different wears in account of their influence on rope strength, total residual endurance of the rope with LMA and LF defects $\eta(x) \geq 0$ can be simply defined by linear superposition

$$\eta(x) = \eta_{LMA}(x) - \chi_{LF}(x),$$

where LF-endurance loss χ_{LF} may be estimated using formula (4) or (5).

More accurate assessment requires data concerning the interference between local faults at different wire layers with respect to friction forces action.

7. Conclusions

1. Statistical simulation allows sufficiently reliable estimates of damaged ropes residual endurance. To elect more correct values of simulating parameters (for example, the initial wires break probabilities), one can use any additional information about rope failure special features.
2. Contribution of localised multiple defects to rope strength degradation is comparable with the contribution of cross-sectional metallic area loss due to missing wires. Both of these factors must be accounted together.
3. Steel rope safety standards related to character and number of wires breaks need refining. Norms must be tightened up in the respect of the number of wire breaks permitted along rope segments, which exceed several lay lengths.
4. To obtain more accurate endurance assessment further information is needed for developing the wear influence functions over the strands cross-sections.

8. References

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